

STATISTICAL INFERENCE

1.0 HYPOTHESIS TESTING

A hypothesis is a statement, inference or tentative explanation about a population that can be tested by further investigation. A hypothesis test is a statistical test that assists in the decision to prove or disprove the statement. In most cases, it may be easier to disprove a hypothesis than to verify it. Sample measurements are taken and inferences made about the total population based on the sample statistics. The inferences and the resulting decisions could be correct or incorrect. In hypothesis testing as well as all other statistical analyses, the risks of making a wrong decision are predetermined.

1.1 Null and Alternate Hypotheses

The symbol μ represents the population or universe mean and \bar{x} represents the sample mean. The symbol σ represents the population standard deviation and σ' or s represent the standard deviation calculated from the sample. The population values μ and σ are parameters and the sample values \bar{x} and σ' or s are statistics.

The following hypothesis is to be tested: The number 5 wheel bearings made by the Canape Bearing Company have an average diameter of 0.65 inches.

Based on the sample statistics, is this statement true or false? The objective of hypothesis testing is to verify the validity of the statement.

The null hypothesis is the hypothesis to be proven or disproven. It is represented by H_0 .

The alternate hypothesis is the hypothesis that is accepted if the null hypothesis is rejected by the test. It is represented by H_1 .

For the test, the hypotheses are stated as follows:

Null Hypothesis, $H_0: \mu = 0.65$
Alternate Hypothesis, $H_1: \mu \neq 0.65$

1.2 Types of Hypothesis Tests

Hypotheses concerning both measurements (variables or continuous data) or counts (attribute or discrete data) may be tested. The hypothesis $H_0: \mu = 0.65$ is an example using continuous data.

A hypothesis concerning discrete data may be stated as follows:

The population from which a sample of one hundred parts was taken has a process average of 2% defective.

$$H_0 : p = .02 \quad \text{or} \quad H_0 : np = 100 \times .02 = 2$$

The symbol μ represents the population mean and \bar{p} represents the sample mean. The symbol np represents the average number of defective parts in the sample for both the hypergeometric and the binomial distributions. For the Poisson distribution, np is the average number of defects in the sample.

In hypothesis testing, a general rule is to use the normal curve areas (Z - statistics) when the sample size n is thirty or more. When n is less than thirty, the t distribution areas (t - statistics) and corresponding tables are used. When n is very large ($n \rightarrow \infty$), the Z table and t table are virtually identical. The method of testing a hypothesis is exactly the same; the only difference is the table that is used.

In the previous chapter $n - 1$ was used instead of n in the denominator of the variance formula. Division by $n - 1$ is called division by degrees of freedom. The term degrees of freedom is so named because only $n - 1$ linear comparisons can be made among n choices. If there are three job candidates and three jobs to fill, there are three choices for the first job. Once it is filled, there are two choices for the second job. After the second job is filled, there is only one choice for the third job. The third job can only be filled by the remaining person, so there are $n - 1$ or two degrees of freedom. To use the t table, the number of degrees of freedom is required.

2.0 DECISION ERRORS

Anytime sample data are used to make decisions or inferences about the entire population, decision errors can be made. In hypothesis testing and acceptance sampling, two types of errors can occur.

Type I error, alpha (α), rejection of a true null hypothesis

Type II error, beta (β), acceptance of a false null hypothesis

	Null hypothesis (H_0) is true	Null hypothesis (H_0) is false
Do not reject H_0	no error ($1 - \alpha$)	Type II error (β)
Reject H_0	Type I error (α)	no error ($1 - \beta$)

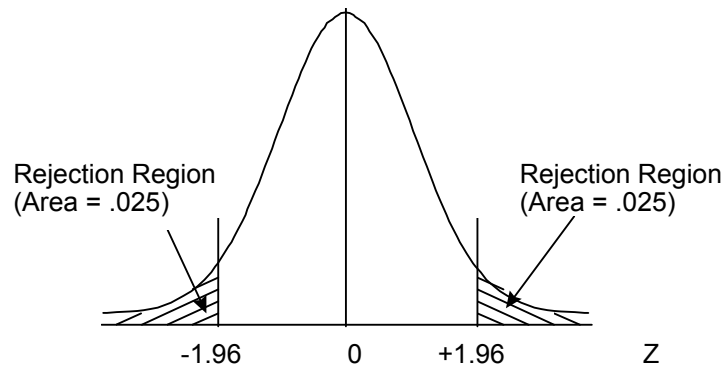
The level of significance is denoted by α . This is the probability of rejecting a null hypothesis when it is true or the probability of making a wrong decision. The confidence interval is denoted by $1 - \alpha$. The confidence interval is the probability of not rejecting a null hypothesis when it is true or the probability of making a right decision. The probability of not rejecting a null hypothesis when it is false is β . This is the probability of making a wrong decision. The term $1 - \beta$ is referred to as the power of test and is the probability of rejecting a null hypothesis when it is false. This is the probability of making a right decision when the null hypothesis is truly false. In acceptance sampling, α and β are referred to as the producer's risk and consumer's risk respectively.

3.0 HOW A HYPOTHESIS IS TESTED

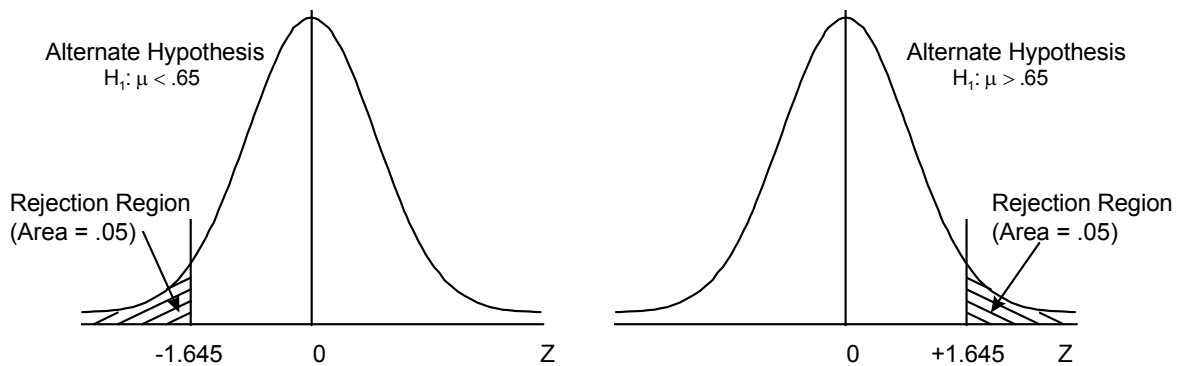
3.1 One or two tailed test?

The alternate hypothesis determines whether the test will be a one tailed test or a two tailed test. If H_1 is stated as $H_1: \mu \neq .65$, then the test is a two tailed test. If H_1 is stated as $H_1: \mu < .65$ or $H_1: \mu > .65$, the test is a one tailed test.

Sketch for a two tailed test ($\alpha = .05$)
Alternate Hypothesis, $H_1: \mu \neq .65$



Sketches for a one tailed test ($\alpha = .05$)



3.2 Examples

Example 1

Example 1 is a step by step procedure on how to perform a hypothesis test. A sample of 100 bearings is taken. The diameter is measured and recorded for each part in the sample. The mean and standard deviation are calculated. The following results are obtained:

$$\bar{x} = 0.67 \text{ inches, } s = 0.08 \text{ inches, } n = 100$$

The distribution of individual data values has a mean of 0.67" and a standard deviation of 0.08". Since a sample mean is to be tested against a population mean, the test distribution is a distribution of averages. The distribution of averages has a mean of 0.67" and a standard deviation or standard error of $0.08/\sqrt{n}$.

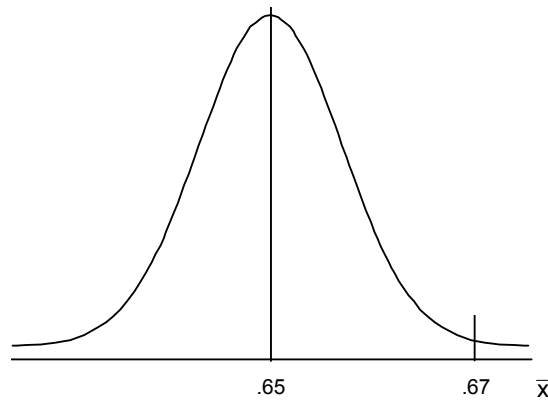
Could the population from which this sample was selected have an average diameter of 0.65 inches? Test this hypothesis against a level of significance of .05 ($\alpha = .05$).

Step 1) State the null and alternate hypotheses.

$$H_0: \mu = 0.65$$

$$H_1: \mu \neq 0.65$$

Step 2) Draw a sketch showing $\mu = .65$ and $\bar{x} = .67$ on the \bar{x} scale.



Step 3) Determine the critical values of Z and the region of rejection.

Since $H_1: \mu \neq .65$, this will be a two tailed test. Half of α will define the rejection region on the left tail of the curve and half of α will define the rejection region on the right tail of the curve. Half of α is $.05/2 = .025$. The sample size is greater than thirty, therefore the correct table to use is the standard normal curve or Z table.

Either draw another sketch or superimpose a Z scale on the first sketch. Look up $.50 - .025 = .475$ in the table. The corresponding Z values are ± 1.96 . These are the critical values. The critical values are the Z values that separate the rejection region from the acceptance region. On the sketch, shade in the areas to the left of -1.96 and to the right of +1.96. The shaded areas indicate the regions of rejection. The area of the acceptance region is .95, therefore this test will have a confidence interval of .95.