

BASIC PROBABILITY

1.0 INTRODUCTION

Probability concepts are familiar to everyone. The weather forecaster states that the probability of rain tomorrow is twenty percent. At the racetrack, the odds are three to one that a certain horse will win the fifth race. Relating probability concepts to manufacturing operations may not be as familiar as the above examples, but they work the same way. Probability is the key to assessing the risks involved in the decision-making process. The gambling casinos determine the probabilities for each game of chance then make the rules so that the odds are always in their favor. The same can be done for manufactured products. The probability of a certain number of defective parts in a large lot can be determined. Also, the percentage of parts within a certain dimension range can be predicted. If the desired results are not obtained, then adjustments to the process can be made. Adjustments to a process in a manufacturing operation are analogous to changing the rules in a casino game. The objective is to obtain the desired results.

Since a major portion of statistical quality control and statistical process control deals with probability concepts, it is important to have a good knowledge of probability. In a manufacturing operation, there are very few occasions when complete information is available. Therefore, information must be generalized from samples and limited known facts. It is sometimes surprising to discover the vast amount of information and knowledge about a process that can be obtained from a relatively small amount of data. Probability is the building block of statistics and statistical quality control.

2.0 EVENTS

An event is defined as any outcome that can occur. There are two main categories of events: Deterministic and Probabilistic.

- A deterministic event always has the same outcome and is predictable 100% of the time.
 - Distance traveled = time x velocity
 - The speed of light
 - The sun rising in the east
 - James Bond winning the fight without a scratch
- A probabilistic event is an event for which the exact outcome is not predictable 100% of the time.
 - The number of heads in ten tosses of a coin
 - The winner of the World Series
 - The number of games played in a World Series
 - The number of defects in a batch of product

In a boxing match there may be three possible events. There could be more events depending on the question asked.

- Fighter A wins
- Fighter B wins
- Draw

2.1 Four Basic Types of Events

- **Mutually Exclusive Events:** These are events that cannot occur at the same time.
The cause of mutually exclusive events could be a force of nature or a man made law. Being twenty-five years old and also becoming president of the United States are mutually exclusive events because by law these two events cannot occur at the same time.
- **Complementary Events:** These are events that have two possible outcomes. The probability of event A plus the probability of A' equals one. $P(A) + P(A') = 1$. Any event A and its complementary event A' are mutually exclusive. Heads or tails in one toss of a coin are complementary events.
- **Independent Events:** These are two or more events for which the outcome of one does not affect the other. They are events that are not dependent on what occurred previously. Each toss of a fair coin is an independent event.
- **Conditional Events:** These are events that are dependent on what occurred previously. If five cards are drawn from a deck of fifty-two cards, the likelihood of the fifth card being an ace is dependent on the outcome of the first four cards.

3.0 PROBABILITY

Probability is defined as the chance that an event will happen or the likelihood that an event will happen.

The definition of probability is

$$\text{Probability} = \frac{\text{Number of Favorable Events}}{\text{Number of Total Events}}$$

The favorable events are the events of interest. They are the events that the question is addressing. The total events are all possible events that can occur relevant to the question asked. In this definition, favorable has nothing to do with something being defective or non-defective.

What is the probability of a head occurring in one toss of a coin?

The number of favorable events is 1 (one head) and the number of total events is 2 (head or tail). In this case, the probability formula verifies what is obvious.

$$\text{Probability of a head} = \frac{\text{Number of Favorable Events}}{\text{Number of Total Events}} = \frac{1}{2}$$

Probability numbers always range from 0 to 1 in decimals or from 0 to 100 in percentages.

3.1 Notation for Probability Questions

Instead of writing out the whole question, the following notation is used.

- What is the probability of event A occurring? = Probability (A) = P(A)
- What is the probability of events A and B occurring? = P(A and B) = P(A) and P(B)
- What is the probability of events A or B occurring? = P(A or B) = P(A) or P(B)

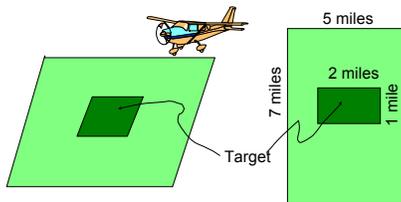
3.2 Probability in Terms of Areas

Probability may also be defined in terms of areas rather than the number of events.

$$\text{Probability} = \frac{\text{Favorable Area}}{\text{Total Area}}$$

Example 1

A plane drops a parachutist at random on a seven by five mile field. The field contains a two by one mile target as shown below. What is the probability that the parachutist will land in the target area? Assume that the parachutist drops randomly and does not steer the parachute.



$$\begin{aligned} \text{Probability of hitting target} &= \frac{\text{Favorable Area}}{\text{Total Area}} \\ &= \frac{2}{35} = .057 \end{aligned}$$

4.0 METHODS TO DETERMINE PROBABILITY VALUES

There are three major methods used to determine probability values.

- **Subjective Probability:** This is a probability value based on the best available knowledge or maybe an educated guess. Examples are betting on horse races, selecting stocks or making product-marketing decisions.
- **Priori Probability:** This is a probability value that can be determined prior to any experimentation or trial. For example, the probability of obtaining a tail in tossing a coin once is fifty percent. The coin is not actually tossed to determine this probability. It is simply observed that there are two faces to the coin, one of which is tails and that heads and tails are equally likely.

- **Empirical Probability:** This is a probability value that is determined by experimentation. An example of this is a manufacturing process where after checking one hundred parts, five are found defective. If the sample of one hundred parts was representative of the total population, then the probability of finding a defective part is .05 (5/100). The question may be asked: How is it known that this sample is representative of the total population? If repeated trials average .05 defective, with little variation between trials, then it can be said that the empirical probability of a defective part is .05.

5.0 MULTIPLICATION THEOREM

The multiplication theorem is used to answer the following questions:

- What is the probability of two or more events occurring either simultaneously or in succession?
- For two events A and B: What is the probability of event A and event B occurring?

The individual probability values are simply multiplied to arrive at the answer. The word “and” is the key word that indicates multiplication of the individual probabilities. The multiplication theorem is applicable only if the events are independent. It is not valid when dealing with conditional events. The product of two or more probability values yields the intersection or common area of the probabilities. The intersection is illustrated by the Venn diagrams in section 11.0 of this chapter. Mutually exclusive events do not have an intersection or common area. The probability of two or more mutually exclusive events is always zero.

For mutually exclusive events:

- $P(A) \text{ and } P(B) = 0$

For independent events:

- Probability (A and B) = $P(A) \text{ and } P(B) = P(A) \times P(B)$

For multiple independent events, the multiplication formula is extended. The probability that five events A, B, C, D and E occur is

$$P(A) \text{ and } P(B) \text{ and } P(C) \text{ and } P(D) \text{ and } P(E) = P(A) \times P(B) \times P(C) \times P(D) \times P(E)$$

Example 2

What is the probability of getting a raise and that the sun will shine tomorrow?

Given: Probability of getting a raise = $P(r) = .10$
 Probability of the sun shining = $P(s) = .30$

The events are independent.

$$P(\text{raise}) \text{ and } P(\text{sunshine}) = P(r) \times P(s) = .10 \times .30 = .03 \text{ or } 3\%$$